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Nonlocal Yukawa Interaction and Fermion Mass-Scale Nonlocality

Kh. Namsrai[1]

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A nonlocal Yukawa interaction between the Higgs boson and the fundamental fermions is introduced. A simple form of this interaction allows us to calculate a particular mass-scale nonlocality for all fundamental fermions. A prediction is given for the mass of the Higgs boson ($m_H \approx 200$ GeV).

The existence of a minimum observable length in string theories (Kato, 1990) and evidence suggesting the existence of fundamental length scales in quantum gravity (Garay, 1994) suggest that the construction of a nonlocal quantum field theory (NQFT) (Namsrai, 1986; Efimov, 1977) that possesses a fundamental scale becomes an attractive and important question (Kleppe and Woodard, 1992; Moffat, 1991; Evans *et al.*, 1991).

Manifestly nonlocal actions for NQFT contain derivatives of infinite order and necessarily contain a scale parameter dimension of length. An old version of these approaches is the Yukawa bilocal or nonlocal fields (Yukawa, 1950).

Recently, the success of the standard model of the electroweak theory has indicated that the Yukawa interaction of Higgs bosons with fundamental fermions plays an important role in understanding the mass scales of fermions and their origin through the Higgs mechanism.

In this paper we study the nonlocal interaction between Higgs bosons and fermions and derive their mass scale of nonlocality.

The question of the mass spectrum of elementary fermions, leptons and quarks, is an important problem in modern physics. Some discussions of this problem can be found in Nambu (1952), Dirac (1962), Fritzsch (1977, 1978),

[1] Institute of Physics and Technology, Mongolian Academy of Sciences, Ulaanbaatar 51, Mongolia.

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Georgi and Jarslkog (1979), Dimopoulos *et al.* (1992a,b), Raby (1992), Sirlin (1994), and Rosen (1995), and references therein. Here we show that in particle physics the masses of elementary fermions are not distributed randomly, but one can predict their mass ratios within the nonlocal method. It turns out that the mass ratios are not dependent on the concrete forms of the Yukawa interaction. We demonstrate this by using a simple scheme where fermions ψ_i ($i = e, \mu, \ldots, u, d, \ldots, t$) interact with the Higgs boson $\varphi_H(x)$ through some nonlocal (Namsrai, 1986; Efimov, 1977) or averaged (Reuter and Wetterich, 1993) interaction of the Yukawa scalar form

$$L_{in}(x) = \frac{1}{2}\Gamma_i \int d^4y_1\, d^4y_2\, K_l(y_1)K_l(y_2)\bar{\psi}_i(x - y_1 - y_2)\psi_i(x - y_1 - y_2) \times [\varphi_H(x - y_1) + \varphi_H(x - y_2)] \tag{1}$$

or in differential form

$$L_{in}(x) = \Gamma_i\bar{\psi}_i(x)\psi_i(x)\phi_H(x) \tag{2}$$

The latter corresponds to the assumption that the Higgs boson carries nonlocality only:

$$\varphi_H(x) \Rightarrow \phi_H(x) = \int d^4y\, K_l(x - y)\varphi_H(y) = K(\Box l^2)\varphi_H(x) \tag{3}$$

where $K_l(x) = K(\Box l^2)\delta^{(4)}(x)$ is the generalized function (Efimov, 1977). For the cases (2) and (3), the propagator of the Higgs boson takes the form

$$D_H(x) = \frac{1}{(2\pi)^4 i}\int d^4p\, e^{-ipx}\frac{V_m(-p^2l^2)}{m_H^2 - p^2 - i\epsilon} \tag{4}$$

where $V_m(z) = K_m^2(z)$, and $K_m(z)$ is the Fourier transform of the generalized function in (3). Without loss of generality we choose a form factor of the type

$$V_m(-p^2l^2) = \exp\left\{-\frac{l^2}{4}(m_H^2 - p^2)\right\} \tag{5}$$

This form factor decreases rapidly in the Euclidean momentum space, the physical meaning of which is that it changes the Yukawa potential at short distances by the formula

$$\frac{g}{4\pi r}e^{-m_H r} \Rightarrow -\frac{g}{8\pi r}\left[2sh m_H r + e^{-m_H r}\phi\left(\frac{1}{2}m_H l - \frac{r}{l}\right) - e^{m_H r}\phi\left(\frac{1}{2}m_H l + \frac{r}{l}\right)\right] \tag{6}$$

where $\phi(x) = (2/\sqrt{\pi})\int_0^x dt\, \exp(-t^2)$ is the probability integral.

The latter is obtained by using the well-known relation

$$U(r) = \frac{g}{(2\pi)^3} \int d^3p \, e^{i\mathbf{pr}} D(\mathbf{p}^2)$$

between the potential (for example, Coulomb or Yukawa) of interacting particles and the propagator of the particle (photon, or scalar particle like the Higgs boson) carrying the interaction between them in the static limit.

Generally speaking, in (1) and (2) the coupling constants Γ_i are arbitrary quantities (Kobayashi, 1995).

Formally, our Lagrangian (2) is reminiscent of the local Yukawa interaction

$$L_{in}^{st}(x) = -\gamma_i \overline{Q}_i(x) Q_i(x) \varphi_{\mathrm{H}}(x) \tag{7}$$

with the coupling constants

$$\gamma_i = g \frac{m_i}{2M_W} \tag{8}$$

for the minimal standard model, where one Higgs doublet is considered and there are no flavor-changing Higgs-mediated interactions.

In (7) and (8), $Q_i(x)$ are the fermion fields and g is the $SU(2)$ gauge coupling constant. For definiteness, we suggest that

$$\Gamma_i = \gamma_i q_i^f \tag{9}$$

where q_i^f are arbitrary quantities. Without loss of generality, in order to obtain a concrete mass scale of nonlocality for fermions, we use the following particular choices:

$$\begin{aligned}
q_{e,\mu}^f &= 1 && \text{for light leptons } e,\ \mu \\
q_\tau^f &= \sqrt{\tfrac{3}{2}} && \text{for } \tau\text{-lepton} \\
q_{u,s,d,c}^f &= \sqrt{2} && \text{for light quarks } u,\ d,\ s,\ c \\
q_b^f &= \sqrt{\tfrac{6}{5}} && \text{for } b\text{-quark} \\
q_t^f &= \frac{3}{\sqrt{5}} && \text{for } t\text{-quark}
\end{aligned} \tag{10}$$

These values give the mass ratios obtained below. It is quite possible that quantum numbers like (10) may be associated with a deeper (super)symmetry which could be broken [for example, $U(1)$ family symmetry] in interactions of the fermions with the Higgs boson. The latter distinguishes them by different values of their "charge" q_i^f. We call it the "mass" charge of fermions.

The next step is to calculate the mass operator for fermions by using the Lagrangian (2) with coupling constants (9) and (10). Since this coupling constant is small, the perturbation theory works very well. A low-order diagram giving the contribution to the mass operator is sketched in Fig. 1.

Thus, the matrix element corresponding to the diagram (Fig. 1) has the standard form

$$\begin{aligned}\Sigma_i(p) &= \frac{\Gamma_i^2}{(2\pi)^4 i}\int d^4p\,\frac{V_M(-k^2l^2)}{m_{\mathrm{H}}^2-k^2}\,\frac{m_i+\hat{p}+\hat{k}}{m_i^2-(p+k)^2}\\ &= -\frac{\Gamma_i^2 m_i}{16\pi^2}\,\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\xi\,\frac{v(\xi)}{\sin\pi\xi}\,(m_i^2l^2)^{\xi}\,\frac{\pi}{\sin\pi\xi}\,\frac{1}{\Gamma(1+\xi)}\,F(\xi, m_i^2, m_{\mathrm{H}})\end{aligned} \tag{11}$$

where

$$F(\xi, m_i^2, m_{\mathrm{H}}^2) = \frac{1}{\Gamma(1-\xi)}\int_0^1 dx\,(1-x)^{-\xi}(1+x)[1+(\delta-2)x+x^2]^{\xi}$$

$$\delta_i^{-1} = \frac{m_i^2}{m_{\mathrm{H}}^2}$$

is the regular function in the half-plane Re $\xi > -1$. In (11) we have used the Mellin representation for (4) and (5) with function $v(\xi) = 2^{-2\xi}/\Gamma(1+\xi)$. After a simple calculation of residues with the assumption $\delta^{-1} << 1$, $m_i^2l^2 < 1$, one gets

$$\delta m_i = m_{0i} - m_i = -\Sigma_i(m) = \frac{3}{16\pi^2}\,\Gamma_i^2 m_i \ln(m_{\mathrm{H}} l\beta_i a) \tag{12}$$

where

$$a = 0.788, \qquad \beta_i = \exp\left[-\frac{1}{18}\,\frac{m_i^2}{m_{\mathrm{H}}^2}\,(1-6\ln\delta_i)\right]$$

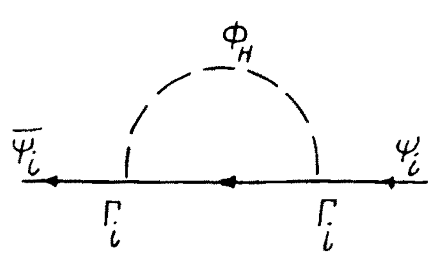

Fig. 1. Diagram of the self-energy of a fermion in the nonlocal Yukawa interaction model.

From (9) and (12) it is obvious that the coefficient of the function

$$\frac{\delta m_i}{m_i^3} = \sigma_i = (q_i^f)^2 \rho \ln(m_{\rm H} l a), \qquad \rho = \frac{3}{16\pi^2} \frac{g^2}{4M_W^2} \tag{13}$$

does not depend on the concrete value of fermion mass and has a universal constant dimension of area. This fact leads to the assumption that equality (13) may be conserved for different fermions with the fixed condition

$$\begin{aligned} m_{\rm H} l &= m_i l_i \qquad \text{(no summation)} \\ l_i &= \Delta / m_i \end{aligned} \tag{14}$$

where Δ is a universal constant within the concrete regularization scheme, i.e., the regularization parameter [or "size" of the $\phi_{\rm H}(x)$ boson in our case] l should be chosen as

$$l = \frac{m_i}{m_{\rm H}} l_i \qquad \text{(no summation)} \tag{15}$$

in the calculation of the quantity σ_i for each fermion case. The constant Δ is chosen in such a way that the mass formula is valid.

In the potential force language, the assumption (15) means that if other particles with mass m_i are near the Higgs bosons, there is a mutual referential mass-dependent force between them due to the potential (6) with the parameter (15) and the change $g \to \Gamma_i$ given by (9). Thus, the assumption

$$2\sigma_i(i = \mu) = \sigma_\tau + \sigma_e \tag{16}$$

with the choice of "mass" charge (10) yields

$$m_\mu / m_e = 3(m_\tau / m_\mu)^{3/2} \tag{17}$$

In this case, the parameter Δ is $\Delta_1 = 1.27$. The equality

$$2\sigma_\mu = \sigma_b + \sigma_e \tag{18}$$

gives

$$m_\mu / m_e = 2(m_b / m_\mu)^{6/5} \tag{19}$$

for which $\Delta = \Delta_2 = 1.269$.

Analogously, the mass ratios

$$\frac{m_\tau}{m_e} = 14\left(\frac{m_t}{m_\tau}\right)^{6/5}, \qquad \frac{m_\mu}{m_u} = 8\left(\frac{m_b}{m_\tau}\right)^{4/5}, \qquad \frac{m_s}{m_u} = 8\left(\frac{m_\tau}{m_s}\right)^{3/4} \tag{20}$$

$$\frac{m_s}{m_d} = 3\left(\frac{m_b}{m_s}\right)^{3/5}, \qquad \frac{m_c}{m_u} = 4\left(\frac{m_t}{m_c}\right)^{9/10}, \qquad \frac{m_\tau}{m_s} = \frac{7}{2}\left(\frac{m_b}{m_\tau}\right)^{4/5},$$
$$\frac{m_c}{m_\mu} = 14\left(\frac{m_\tau}{m_c}\right)^{3/4} \tag{21}$$

follow from the equations

$$2\sigma_\tau = \sigma_t + \sigma_e, \qquad 3(\sigma_\mu - \sigma_u) = 2(\sigma_b - \sigma_\tau), \qquad 2\sigma_s = \sigma_\tau + \sigma_u \tag{22}$$

$$2\sigma_s = \sigma_b + \sigma_d, \qquad 2\sigma_c = \sigma_t + \sigma_u, \qquad 2\sigma_\tau = \sigma_b + \sigma_s, \qquad 2\sigma_c = \sigma_\tau + \sigma_\mu \tag{23}$$

respectively. For all these relations universal constant is

$$\Delta \sim 1/a = 1.27 \tag{24}$$

which provides assumptions (14) and (15), as it should.

The ratios (17) and (19)–(21) are calculated by knowing only the mass of two particles, say the electron and muon. Why do two electrons exist in nature? The simple answer is that both are needed in the first cycle of mass hierarchy to set up the whole mass spectrum of the fundamental fermions. They are shown in Table I (in MeV).

Comparing these quantities with lepton and quark mass listings (Review of Particle Properties, 1994), we observe full agreement with the standard theory and experiments. For example, while the experimental value is (Abe *et al.*, 1995; Abachi *et al.*, 1995) $m_t = 176 \pm 8$ GeV, in our case we have 175.9 GeV, and while a recent computation gives a b-quark pole mass of 4.94 ± 0.15 GeV, in our case we have 5.042 GeV. From the heavy quark-effective theory (HQET) it follows that $m_b - m_c = 3.4$ GeV, while the mass ratios give 3.455 GeV. In lowest order of the chiral perturbation theory $m_u/m_d = 0.56$ and $m_s/m_d = 20.1$, while in our case we find 0.51 and 19.7, respectively. The quark masses for light quarks u, d, and s discussed so far are often referred to as current quark masses and the HQET mass is not the same as the pole mass.

Table I. Fermon Masses (MeV)

m_e	m_μ	m_τ	m_u	m_d	m_s	m_c	m_b	m_t
0.511	105.658	1776.1	5.7	11.2	219.8	1587.0	5042.1	175,895.6

Now let us consider the standard model in which the fermions form three families: ν_e, e, u, and d; ν_μ, μ, c, and s; and ν_τ, τ, t, and b. The left-handed quarks are grouped into $SU(2)$ doublets,

$$Q_{aL} \equiv \begin{pmatrix} u_a \\ d_a \end{pmatrix}_L$$

where a labels the family ($a = 1, 2, 3$). The right-handed quarks are $SU(2)$ singlets, u_{aR} and d_{aR}. The left-handed leptons are grouped in a doublet

$$L_{aL} = \begin{pmatrix} \nu_a \\ e_a \end{pmatrix}_L$$

The right-handed electron e_{aR} is an $SU(2)$ singlet. Gauge-invariant Yukawa couplings such as

$$h^d_{ab}\overline{Q}_{aL}\phi d_{bR} + h^e_{ab}\overline{L}_{aL}\phi e_{bR} + \text{h.c.} \tag{25}$$

can be introduced to give masses to the charge-1/3 quarks and the leptons. The term

$$h^u_{ab}\overline{Q}_{aL}\phi_c u_{bR} + \text{h.c.} \tag{26}$$

gives the mass formula to the charge-2/3 quarks. In (25) and (26) ϕ and ϕ_c are the Higgs doublet scalars. In the standard model, the self-energy diagram of Fig. 1 takes the form (say, for the t-quark) shown in Fig. 2.

Matrix elements corresponding to these diagrams (Fig. 2) acquire the form

$$\Sigma^b_t(p) = \frac{g^2_{btH}}{(2\pi)^4 i}\int d^4p\, \frac{V_m(-k^2l^2)}{m^2_{\rm H} - k^2 - i\epsilon}\,\frac{1+\gamma_5}{2}\,\frac{m_b + \hat{p} + \hat{k}}{m_b^2 - (p+k)^2 - i\epsilon}\,\frac{1+\gamma_5}{2} \tag{27}$$

and

$$\Sigma^t_t(p) = \frac{g^2_{ttH}}{(2\pi)^4 i}\int d^4p\, \frac{V_m(-k^2l^2)}{m^2_{\rm H} - k^2 - i\epsilon}\,\frac{1+\gamma_5}{2}\,\frac{m_t + \hat{p} + \hat{k}}{m_t^2 - (p+k)^2 - i\epsilon}\,\frac{1+\gamma_5}{2} \tag{28}$$

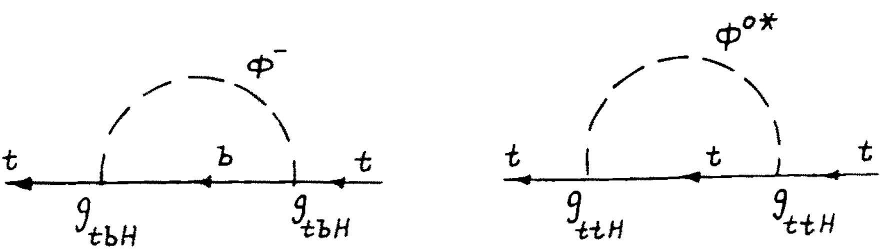

Fig. 2. Self-energy diagram for the quark in the standard model.

Here we have used the nonlocal regularization method (Fritzsch, 1977, 1978; Georgi and Jarlskog, 1979) and g_{btH} and g_{ttH} are coupling constants of the vertexes $\bar{u}_t u_b \phi^-$ and $\bar{u}_t u_t \phi^{0*}$, respectively. Simple calculation yields a mass contribution to the t-quark:

$$\begin{aligned}\delta m_t^{st} &= \bar{u}_t^L[\Sigma_t^b(m_t) + \Sigma_t^t(m_t)]u_t^R + \text{h.c.}\\ &= \frac{(q_t^f)^2}{16\pi^2}\rho^{st}\ln(m_H a^{st}\beta^{st}l)\\ &\quad\times\{g_{btH}^2 m_b/(q_t^f)^2\rho^{st} + g_{ttH}^2 m_t/(q_t^f)^2\rho^{st}\}\end{aligned} \tag{29}$$

if and only if the ratio

$$\frac{m_t}{m_H\epsilon} = \left(\frac{m_H\epsilon}{m_b}\right)^{g_{ttH}^2 m_b/g_{btH}^2 m_t}, \qquad \epsilon = 0.7788 \tag{30}$$

is valid. Here

$$a^{st} = 0.667$$

$$\beta_t^{st} = \exp\left\{-\frac{1}{4}\frac{m_b^2}{m_H^2}(1 - 2\ln\delta_b) - \frac{1}{4}\frac{m_t^2}{m_H^2}(1 - 2\ln\delta_t)\right\}$$

$$\rho^{st} = \frac{1}{3}\rho$$

and ρ is given in (13).

Now we redefine the quantity (13) in the standard model as

$$\begin{aligned}\sigma_t^{st} &= \delta m_t^{st}/\{g_{btH}^2 m_b/(q_t^f)^2\rho^{st} + g_{ttH}^2 m_t/(q_t^f)^2\rho^{st}\}\\ &= \frac{(q_t^f)^2}{16\pi^2}\rho^{st}\ln(m_H a^{st}\beta_t^{st}l)\end{aligned} \tag{31}$$

Analogous calculations for other fermions are carried out, which lead to the redefinition of the corresponding quantities σ_i^{st}. Putting σ_i^{st} into equations (16), (18), (22), and (23) with the condition (14), where

$$\Delta \to \Delta^{st} \sim 1/a^{st} = 1.5 \tag{32}$$

one can obtain the same mass ratio formulas as in (17) and (19)–(21). The result is not changed in the standard model because the structure of the logarithm function in (29) and (31) is also conserved in this case.

It is evident that powers and numerical coefficients (as parameters of the theory) in relations (17) and (19)–(21) are defined uniquely by ratios of "mass" charges (10) and by conditions (14) and (15) using (24) and (32) and recent theoretical results and experimental data on fermion masses.

We see that the mass ratios are scaling invariant with respect to the transformation $\Gamma_i \to \Gamma_i' = \lambda\Gamma_i$, where λ is an arbitrary number, and that equalities (24) and (32) minimalize expressions (12) and (29). The latter means that mass values of fermions in the Lagrangian of the theory are "pole" (or potential) ones and the mass ratios obtained for them are valid.

Now we calculate the contribution to the t-quark mass ratios in (20) and (21) due to the Higgs boson mass by formulas (12) and (31). This contribution changes the mass of the t-quark as

$$m_t(\delta = 0) \Rightarrow m_t(\delta) = m_t(0) \exp\left\{-\frac{1}{18}\frac{m_t^2(0)}{m_H^2}(1 - 6 \ln \delta_t)\right\} \tag{33}$$

Such a change leads to the mass ratio, for example,

$$\frac{m_\tau}{m_e} = 14\left[\frac{m_t(\delta)}{m_\tau}\right]^{6/5} = 14\left[\frac{m_t^{\exp}}{m_\tau}\right]^{6/5}\left\{\frac{m_t(0)}{m_t^{\exp}} \exp\left[-\frac{1}{16}\frac{m_t^2(0)}{m_H^2}(1 - 6 \ln \delta_t)\right]\right\}^{6/5}$$

The experimental value (Abe *et al.*, 1995) of $m_t^{\exp} = 176 \pm 8$ GeV almost coincides with $m^t(0)$ shown in Table I, and therefore

$$m_t^{\exp} = 176 \pm 8 = m_t(0)\left[1 - \frac{1}{18}\frac{m_t^2(0)}{m_H^2}(1 - 6 \ln \delta_t)\right] \tag{34}$$

from which it follows that

$$m_H = 211 \text{ GeV} \tag{35}$$

and

$$m_H = 246 \text{ GeV} \tag{36}$$

for the cases (12) and (31), respectively.

From (30) it follows that the value of (35) gives $r = g_{ttH}/g_{tbH} \sim 4/5$. If $r = 1$, then $m_H = 205$ GeV. Moreover, the ratio (30) yields the upper bound $m_H \lesssim 226$ GeV.

Note that one can derive the mass ratios (17) and (19)–(21) in the more general case when the coupling constants of the Yukawa interaction (2) or (25) and (26) are arbitrary, as long as there exist some relationship between them. For instance, the mass ratio equation $\delta m_i/m_i - \delta m_k/m_k = \delta m_j/m_j - \delta m_i/m_i$ for any three fermions i, j, k using (12) and conditions (14) and (15) decomposes into two parts:

$$m_i/m_k = c(m_j/m_i)^{\Gamma_j^2/\Gamma_i^2}$$

$$m_i/m_k = c(m_j/m_i)^{\Gamma_j^2/\Gamma_i^2}(\Delta a)^{2-\Gamma_k^2/\Gamma_i^2-\Gamma_j^2/\Gamma_i^2}$$

Assuming $i = \mu$, $k = e$, $j = \tau$, $\Delta a = 1$, $c = 3$, and $\Gamma_\tau^2/\Gamma_\mu^2 = 3/2$, we get the ratio (17) and so on. These equations are universal. In particular, even if all the coupling constants Γ_i are the same, the mass ratios (17) and (19)–(21) without any powers are valid with different coefficients: 37/3, 13/3, 35, 20/3, 4, 7/9, 8/3, 3, and 41/3, respectively. In this case, the masses of the fundamental fermions are close to those shown in Table I, namely 0.511, 105.658, 1771.4, 5.6, 10.1, 198.6, 1614.1, 5041.6, and 175439.3 (in MeV). Thus the universality of the mass scales of nonlocality is preserved for all cases due to the presence of the scale parameter l in the nonlocal action (2) characterizing the mutual referential mass-dependent potential (6) and satisfying conditions (14) and (15).

Finally, it should be noted that in equations (20) and (21) we compare lepton and quark masses while ignoring the strong scale dependence of the quark masses in QCD. This problem will be considered elsewhere.

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